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Coherent resonant tunnelling in an impurity quantum well in the presence of electric and strong magnetic fields

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Abstract. A further development is described of an analytical approach to the problem of an impurity electron (or hole) in a quantum well (QW) subject to electric and strong magnetic external fields both directed perpendicular to the hetero-planes. Previous results for an impurity centre at the edge of a QW are extended to the case in which the impurity is located at any position in the QW. It is shown that, as in the edge case, the combined potential acting on the electron (or hole) resembles that of a double quantum well. One effective well is formed by the Coulomb potential and the QW boundary closest to the impurity. A second effective well is formed by the electric field potential and the other boundary of the QW. Analytical expressions for the energy levels of the impurity are obtained. When the levels associated with the two effective QWs anti-cross, the impurity single QW can be treated as a resonant structure. The explicit dependencies of the resonant splitting upon the width of the QW, the magnitudes of the electric and magnetic fields and the position of the impurity centre are obtained. Estimates of the expected splitting and frequency of the emitted radiation relevant to the inter-well oscillations of the electron are made using typical values associated with GaAs QWs.

1. Introduction

During the last decade, the subject of resonant devices based on low-dimensional heterostructures has been studied extensively both experimentally and theoretically. A study of impurities in low-dimensional structures is important on at least two accounts: in basic research and in their impact on the fundamental and applicable properties of the heterostructures. Obtaining the wealth of experimental data has been possible on account of the advances made in growth techniques and control by the development of molecular-beam epitaxy (MBE) and metal–organic chemical vapour deposition (MOCVD) methods of manufacturing particular structures. Much of the more recent work has concentrated on a double-quantum-well (DQW) structure formed by two narrow-gap semiconductor layers separated by a wide-gap barrier layer. Most of the interest is the resonant tunnelling between the wells. Quantum Hall states, cyclotron resonance, magneto-plasmon excitations and magneto- and electro-transport are all affected strongly by the resonance process. Resonant inter-well oscillations lead to the emission of terahertz radiation from DQW structures. This in turn provides the basis for high-frequency devices such as blue-light-emitting diodes and laser diodes and also for multivalent logic applications in integrated circuits.

Resonance effects in a DQW structure are closely connected with the problem of impurities in a single quantum well (SQW) in the presence of electric and magnetic fields. However, the resonance condition for a real DQW requires accurate knowledge of values

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for the parameters associated with the well and barriers. This implies accurate control of the MBE or MOCVD growth technique so that gradual changes in these parameters can be incorporated. If the DQW is constructed from an impurity in a SQW together with an applied electric field, the resonance condition can be provided by a gradual change in the magnitude of the electric field. This condition is simpler to impose experimentally.

The effects of electric and magnetic fields acting on heterostructures containing impurities have been discussed in numerous experimental and theoretical papers during the last decade. For example, comprehensive summaries have been given by Santiago *et al* [1] and Shi *et al* [2] concentrating on SQW structures. Earlier, Greene and Bajaj [3, 4] and Greene and Lane [5] used variational methods to study the effects of magnetic fields whilst the same techniques were used later by Cen and Bajaj to consider the impurity states in symmetric [6] and asymmetric [7] QWs subject to parallel electric and magnetic fields directed perpendicular to the hetero-planes.

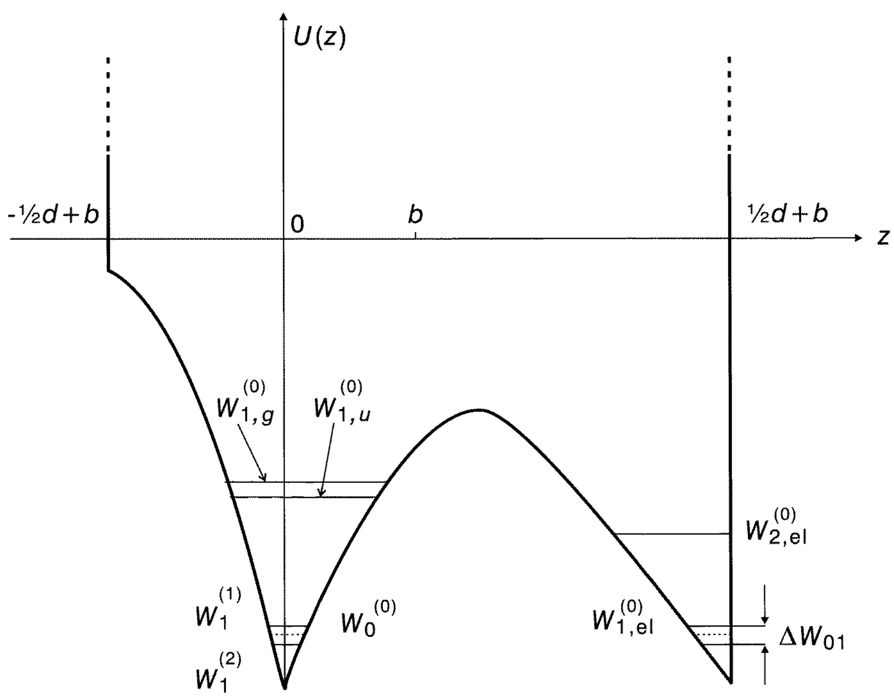

Figure 1. The schematic form of the potential $U(z) \simeq -e^2/4\pi\varepsilon_0\varepsilon|z| - eEz$, where $W^{(0)}_{1g,u}$ (from equation (3.8)) and $W^{(0)}_{2,el}$ (from equation (3.11)) are the first excited quasi-even (g) and quasi-odd (u) quasi-Coulomb and 'electric' levels respectively. The electric field E and the QW width d are chosen to provide resonance between the ground quasi-Coulomb level $W_0^{(0)}$ (equation (3.7)) and 'electric' level $W^{(0)}_{1,el}$ (equation (3.11)). ΔW_{01}, given by equation (3.18), is the resonance splitting of these levels.

Whilst the majority of papers describe results obtained using numerical techniques, alternative analytical methods are also of much interest as they keep the basic physics of the problem in view. Analytical work has been used by the current authors previously for the problem of impurity states in a QW subject to a strong magnetic field [8, 9]. The effect of parallel electric and strong magnetic fields on the impurity states in a QW has also been studied analytically [10, 11] assuming the QW to be narrow compared to the radius of the impurity electron state. It was noted that these impurity states resemble size-quantized states whilst the presence of the electric field removed inversion symmetry from the problem.

Very recently [12], the authors have extended the analytical method to the case in which the impurity is located at the edge of the SQW. In such a case, the combined potential acting on the electron resembles that of a double QW. It was shown that such a system can be treated as if it is a resonance structure. In the present paper, the problem of an impurity positioned anywhere between the mid-point and the edge of a SQW in the presence of parallel electric and strong magnetic fields is developed. The fields are directed perpendicular to the layers. As in reference [12], the width of the QW is taken to be greater than the effective Bohr radius associated with the impurity electron, so the electron states have a quasi-Coulomb character. This is because the combined potential governing the impurity electron states in the SQW has the appearance of a DQW, as shown schematically in figure 1. One part is formed from the one-dimensional Coulomb potential and the QW boundary close to which the impurity is located and the other part is formed from the electric field potential and the other boundary of the QW. The explicit dependencies of the energies of the impurity electron upon the magnitudes of the magnetic and electric fields, the width of the QW and position of the impurity are obtained.

The theory presented in this paper is a non-trivial extension of that given originally in reference [12] for an impurity at the edge of a QW. The analysis presented focuses on the resonance between the energy levels associated with the two effective wells. Tunnelling of the electron through the barrier separating the effective wells becomes possible and anti-crossing of the energy levels can occur. Simple analytical expressions for the resonance splitting of the levels ΔW are derived. The gap between the resonance levels defines the tunnelling time $\pi\hbar/\Delta W$ and results in an alteration in the spatial distribution of the wave function. This in turn defines the kinetic and optical properties and particularly the frequency of the radiation $\omega = \Delta W/\hbar$ emitted by such semiconductor structures. Estimates for the expected energy gap ΔW and for the frequency ω are made for GaAs QWs.

2. General theory

The underlying theory to be used here was developed originally in references [9], [10] and [12]. It is based on the usual effective-mass approximation. The z-axis is chosen to lie along the direction of the parallel uniform magnetic ($\boldsymbol{B}$) and electric ($\boldsymbol{E}$) fields which are perpendicular to the hetero-planes. The QW is modelled by an infinite square well of width d. This analysis concentrates on the problem in which the impurity is positioned anywhere between the edge and mid-point of the QW. The positions of the impurity and the mid-point of the QW are taken to be the points $z = 0$ and $z = b$ respectively (figure 1). We assume further that the energy bands are non-degenerate and spherically symmetric with a parabolic cross section.

In cylindrical coordinates, the effective-mass equation has the form

$$\left(\frac{1}{2\mu}\left(-\mathrm{i}\hbar\boldsymbol{\nabla} + \frac{1}{2}e(\boldsymbol{B}\times\boldsymbol{r})\right)^2 - \frac{e^2}{4\pi\varepsilon_0\varepsilon\sqrt{\rho^2+(z-b)^2}}\right)\Psi(\rho, z) = E\Psi(\rho, z) \tag{2.1}$$

where ε_0 is the permittivity of free space, ε is the dielectric constant and μ and e are the effective mass and charge of the carrier respectively. The energy E and wave function $\psi(\rho, z)$ can be found by solving this equation subject to the boundary conditions

$$\Psi\left(\rho, \pm\frac{1}{2}d\right) = 0. \tag{2.2}$$

The characteristic dimensionless parameters of the problem are the impurity radius a_0 $(=4\pi\overline{h}^2\varepsilon\varepsilon_0/\mu e^2)$, the magnetic length a_B $(=(\overline{h}/eB)^{1/2})$ and the width d of the QW. In the limit of a strong magnetic field (for which the effect of the magnetic field on the electron at a position $\boldsymbol{r}(\rho, z)$ considerably exceeds that of the Coulomb field of the impurity centre such that $a_B \ll a_0$), the motions of the electron along the z-direction and in the x–y plane can be separated. Thus the solution to (2.1) may be written in the form

$$\Psi_{N,m}(r) = X_{\perp N,m}(\rho) f^{(N,m)}(z) \tag{2.3}$$

where the function $X_{\perp N,m}(\rho)$ describes the motion of the electron in the magnetic field in the x–y plane. The labels N and m are quantum numbers such that $N = 0, 1, 2, \ldots$ and $m = 0, \pm 1, \pm 2, \ldots$. In order to simplify the calculations, we consider only the ground transverse state for which $N = m = 0$ which is given by the function $X_{\perp 0,0}(\rho) = \chi_\perp(\rho)$, although the results obtained below are valid qualitatively for any transverse state.

It has been found [8, 9] that the levels of lowest energy associated with the longitudinal motion have a quasi-Coulomb character for a wide quantum well. These levels are affected strongly by external fields whilst the highest-energy size-quantized levels depend only weakly upon both the magnitudes of the fields and the impurity potential. Further, we consider the quasi-Coulomb states having longitudinal energies $W < 0$.

The longitudinal function $f_\lambda(u)$ of energy E_λ, for the case in which the electric field is directed along the negative z-direction, satisfies the equation [12]

$$\frac{\mathrm{d}^2 f_\lambda(u)}{\mathrm{d}u^2} + \left(\lambda\langle 0|(u^2+g^2)^{-1/2}|0\rangle + \frac{\lambda^3}{8} su - \frac{1}{4}\right) f_\lambda(u) = 0 \tag{2.4}$$

where $\langle 0|\cdots|0\rangle$ is an average with respect to the function $\chi_\perp(\rho)$. Also,

$$u = 2z/a_0\lambda \qquad g = 2\rho/a_0\lambda \qquad W_\lambda = -R/2\lambda^2 \tag{2.5}$$

where $R = e^2/4\pi\varepsilon_0\varepsilon a_0$ is the impurity Rydberg constant. The quantum number λ labels the states of the motion along the z-axis as in reference [12]. The other parameter relevant to the calculation is the dimensionless electric field $\boldsymbol{s}$ (which is the electric field $\boldsymbol{E}$ scaled relative to the impurity electric field). It is defined [12] by

$$s = \frac{2E}{e/(4\pi\varepsilon_0\varepsilon a_0^2)}. \tag{2.6}$$

The boundary conditions corresponding to the right-hand and left-hand edges of the QW respectively have the form

$$f[(d/a_0\lambda)(2b/d \pm 1)] = 0. \tag{2.7}$$

The analysis of equation (2.4) will be based upon the Hasegawa–Howard method [13] together with a comparison equation [14]. Our approach to the problem is to consider the solution to equation (2.4) for the three regions in turn and match them at the boundaries.

2.1. Matching the three regions

2.1.1. The region containing the impurity. Following the approach developed in references [8] and [12] under the condition

$$|u| \gg \langle 0|g|0\rangle \sim 2a_B/a_0\lambda_0 \tag{2.8}$$

the general solution to equation (2.4) is, to a first approximation (for which $s = 0$),

$$f_\lambda(u) = A_\pm W_{\lambda,1/2}(|u|) + B_\pm M_{\lambda,1/2}(|u|) \tag{2.9}$$

where $W_{\lambda,1/2}$ and $M_{\lambda,1/2}$ are Whittaker functions. Also, $A_\pm$ and $B_\pm$ are constants relevant to the regions $u > 0$ and $u < 0$ respectively.

In the region $|u| \ll 1$, an iteration method is performed by the double integration of (2.4) using the trial function

$$f_\lambda^{(0)}(u) = c_\pm + \alpha_\pm |u|(u^2 + g^2)^{1/2} \ln[|u| + (u^2 + g^2)^{1/2}]. \tag{2.10}$$

The constants $c_\pm$ and $\alpha_\pm$ correspond to the regions $u > 0$ and $u < 0$ respectively. The results of the integration for the region $|u| \gg \langle 0|g|0\rangle$ and from the standard expansion of the Whittaker functions involved in (2.9) for $|u| \ll 1$ (see, for example, Gradshteyn and Ryzhik [15]) are compared. When terms of the same order are equated, a set of four linear equations are obtained. The continuity conditions applied to the function $f_\lambda^{(0)}(u)$ given in (2.10) and its first derivative at $u = 0$ give the results $c_+ = c_-$ and $\alpha_+ = -\alpha_-$.

2.1.2. The region adjacent to the right-hand boundary of the QW. A comparison equation method [14] will be used. In this region, the effect of the electric field $\boldsymbol{E}$ overcomes the influence of the impurity centre, so the comparison equation in place of (2.4) is the equation for the Airy functions Ai and Bi [16]. The general solution to this equation can be written in the form

$$f_\lambda(u) = J\,\mathrm{Ai}(-\eta) + K\,\mathrm{Bi}(-\eta) \tag{2.11}$$

where J and K are constants. Also,

$$\eta(u)^{3/2} = \frac{3}{2}\int_{t_1}^{u} q(t)\,\mathrm{d}t \qquad \text{for } \eta > 0 \tag{2.12}$$

and

$$|\eta(u)|^{3/2} = -\frac{3}{2}\int_{t_1}^{u} |q(t)|\,\mathrm{d}t \qquad \text{for } \eta < 0 \tag{2.13}$$

where

$$q(t) = [\lambda/t - \tfrac{1}{4} + (\lambda^3/8)st]^{1/2} \tag{2.14}$$

and where t_1 is the greater root of the equation $q(t_1) = 0$.

2.1.3. The intermediate region. Within the region defined by $u \gg 1$ and $(\lambda^3/8)su \ll \lambda/u$ and under the condition $s\lambda^4 \ll 1$, the functions (2.9) and (2.11) can be matched. A comparison of the expression given in equation (2.9) for $u \gg 1$ [15] and that obtained from equation (2.11) for the asymptotic region $\eta < 0$ with $|\eta| \gg 1$ [16] is then made. When terms of the same form are equated, a set of two linear equations is obtained.

2.2. An analysis of the set of equations

The set of six linear algebraic equations above for the coefficients $A_\pm$, $B_\pm$, $c_\pm$, $\alpha_\pm$, K and J together with the boundary conditions (2.7) and the requirements $c_+ = c_-$ and $\alpha_+ = -\alpha_-$ are solved by the determinantal procedure. This gives a transcendental equation in the form

$$\left\{-\frac{W}{M\lambda\Gamma^2(-\lambda)}\left[\varphi + \frac{1}{2}\xi - |Q|\right] + \frac{1}{\lambda\Gamma^3(-\lambda)}[\varphi + \xi][\varphi - 2|Q|]\right\}\mathrm{Ai}(-\eta_1)$$
$$+\frac{1}{2}\mathrm{e}^{-2\Phi}\,\mathrm{Bi}(-\eta_1)\left\{\frac{1}{\Gamma(-\lambda)}\left[\varphi + \frac{1}{2}\xi - |Q|\right] - \frac{W}{M}\right\} = 0. \tag{2.15}$$

In the above expression, the functions $\varphi(\lambda)$, $\xi(\lambda)$ and $\Phi(\lambda)$ are given by

$$\varphi(\lambda) = \psi(1-\lambda) + 1/(2\lambda) + 2C - 1 \tag{2.16a}$$

$$\xi(\lambda) = 1 - \tfrac{1}{2}C + \ln(\gamma/\lambda) \tag{2.16b}$$

and

$$\Phi(\lambda) = 2/[3s\lambda^3] + \lambda \ln(s\lambda^3/8) \tag{2.16c}$$

where $\gamma = 2^{1/2}a_B/a_0 \ll 1$, $\Gamma(x)$ is the gamma function, $\psi(x)$ is the psi function (the logarithmic derivative of the gamma function) and C is the Euler constant (=0.577). Also, we define

$$Q(\lambda) = \frac{2\langle 0|g\ln g|0\rangle}{\lambda\langle 0|g^2|0\rangle} = \frac{\sqrt{\pi}}{2\gamma}\left(\ln\frac{\gamma}{\lambda} + 1 - \tfrac{1}{2}C\right) \qquad \text{for } Q(\lambda) < 0 \tag{2.17}$$

where $W \equiv W_{\lambda,1/2}(u_2)$, $M \equiv M_{\lambda,1/2}(u_2)$ and $u_{1,2} = (d/a_0\lambda)(1 \pm 2b/d)$. On solving equation (2.15), the quantum number λ can be found which in turn determines the impurity electron energy W_λ.

Equation (2.15) is valid in the strong-magnetic-field region ($\gamma \ll 1$) and under the condition $u_2 \gg \gamma/\lambda$. The latter condition implies that the approach given here is applicable only to cases in which the impurity is separated from the left-hand edge of the QW by a distance greater than the magnetic length a_B. Similarly, the condition $Q < 0$ (see equation (2.17)) is also valid for a sufficiently strong magnetic field. In parallel with these restrictions, equation (2.15) remains valid over large regions within the wide QW of width d and the impurity centre position b and in cases in which the electric field E is less than the impurity electric field.

As expected, equation (2.15) satisfies the limiting case of zero electric field. Setting $E = 0$ in (2.15) and using asymptotic expressions [16] for the Airy functions $\mathrm{Ai}(-\eta_1)$ and $\mathrm{Bi}(-\eta_1)$ for large values of $|\eta_1| \gg 1$ with $\eta_1 < 0$ we find that the equation for the quantum numbers λ coincides with that obtained previously for the quantum numbers of a diamagnetic impurity centre positioned anywhere in the QW and in the absence of an electric field [8].

3. Results

As stated previously, the main aim of this approach is to investigate resonances between states formed within the two effective potential wells formed by the Coulomb impurity potential and the left-hand boundary of the QW, and by the uniform electric field E and the right-hand boundary of the QW. They contain Coulomb and so-called 'electric' levels respectively as given by the first and second factors in the first term in the left-hand part of equation (2.15). The last term in the left-hand part of this equation describes the tunnelling of an electron from the impurity well towards the triangular well close to the right-hand boundary through the potential barrier which has a power [12] $\Phi \gg 1$.

3.1. The zeroth approximation

3.1.1. The quasi-Coulomb states. Neglecting the tunnelling term as an approximation, equation (2.12) decomposes into the two independent equations

$$\frac{1}{\Gamma(-\lambda)}(\varphi + \xi)(\varphi - 2|Q|) - \frac{W}{M}\left(\varphi - |Q| + \frac{1}{2}\xi\right) = 0 \tag{3.1a}$$

and

$$\mathrm{Ai}(-\eta_1) = 0 \tag{3.1b}$$

which arise from the two effective wells. Equation (3.1) describes the ground and first excited impurity levels in the quasi-Coulomb well adjacent to the left-hand boundary. In the absence of the electric field and when the impurity centre is situated at the centre of the QW or at any position within a bulk semiconductor, the electron states have a definite parity [8, 9]. However, the classification of the energy levels into two groups can be made for the impurity centre at any position in the well; all states are referred to as quasi-even or quasi-odd states. To simplify the explicit expressions for the energy levels, we assume that the effect of the strong magnetic field on the one-dimensional Coulomb states is larger than that of the finite width of the QW. The relevant conditions will be given below. For the wide QW ($d \gg a_0$) the asymptotic expansion [15] of the Whittaker functions at $u_2 \gg 1$ will be used.

(a) *The quasi-even states.* It follows from (3.1) that, in this case, the equation for the quasi-even states can be written in the form

$$\varphi = -\xi - \tfrac{1}{2}\lambda\Gamma^2(-\lambda)G_\lambda \tag{3.2}$$

where $G_\lambda = \exp(-u_2 + 2\lambda \ln u_2) \ll 1$. The quantum number λ for quasi-even states is given by $\lambda_{n\mathrm{g}} = n + \delta n_\mathrm{g}$ where $n = 0, 1, 2, \ldots$ and where $\delta n_\mathrm{g} < 1$. For the ground level ($n = 0$) on using $\lambda\Gamma^2(-\lambda) = \delta_0^{-1}$ and in the logarithmic approximation ($\gamma \ll 1$, $|\ln\gamma| \gg 1$) we obtain from (3.2)

$$\delta_0 = (-2\ln\gamma - C)^{-1}(1 + G_\lambda). \tag{3.3}$$

For the excited states ($n = 1, 2, \ldots$) and using $\lambda\Gamma^2(-\lambda) = n(n!)^{-2}(\delta n_\mathrm{g})^{-2}$ we have

$$\delta n_\mathrm{g} = -[\ln(\gamma/n) + 3C/2 + 1/(2n) + \psi(n)]^{-1} + (n/2(n!)^2)G_n. \tag{3.4}$$

(b) *The quasi-odd states.* It follows from (3.1) that the equation for the quasi-odd states can be written in the form

$$\varphi = 2|Q| - \tfrac{1}{2}\lambda\Gamma^2(-\lambda)G_\lambda. \tag{3.5}$$

The quasi-odd states have a quantum number λ given by $\lambda_{n,\mathrm{u}} = n + \delta n_\mathrm{u}$, where $n = 1, 2, \ldots$ and where $\delta n_\mathrm{u} < 1$. Similarly, as in the case of the quasi-even states, on solving equation (3.5) we have

$$\delta n_\mathrm{u} = (2|Q| - \psi(n) - 2C + 1 - 1/(2n))^{-1} + (n/2(n!)^2)G_n. \tag{3.6}$$

The expressions (3.3), (3.4) and (3.6) are valid under the condition

$$G_\lambda^2\lambda^2\Gamma^4(-\lambda)\gamma^2\ln^{-2}\gamma \ll 1.$$

Thus the impurity levels consist of the non-degenerate ground level ($n = 0$) and excited levels ($n = 1, 2, \ldots$) having a doublet structure consisting of quasi-even and quasi-odd components.

(c) *The impurity Stark effect.* The effect of the electric field directed parallel to a strong magnetic field on the quasi-Coulomb states was considered in references [17] and [18]. For the ground level ($n = 0$) the electric field leads to a decrease in energy $\sim E^2$, so

$$2\frac{W_0^{(0)}}{R} = -\frac{1}{\delta_0^2}\left(1 + \frac{5}{16}s^2\delta_0^6\right) \tag{3.7}$$

where δ_0 is given by (3.3). The expressions for the excited states will be given below for the case in which the splitting of the doublet by the electric field exceeds the initial splitting in the case for $E = 0$. Thus

$$\frac{2W_\lambda^{(0)}}{R} = -\frac{1}{\lambda_{n\mathrm{g,u}}^2}\left(1 \mp \frac{3}{2}s\lambda_{n\mathrm{g,u}}^4\right). \tag{3.8}$$

The negative and positive signs inside the bracket correspond to the quasi-even and quasi-odd components respectively.

3.1.2. The 'electric' levels. The second of equations (3.1) describes the ground and nearest so-called 'electric' levels in the triangular well adjacent to the right-hand boundary of the QW. The solution to this equation has the form [16] $\eta_1 = \alpha_k$, $k = 1, 2, 3, \ldots$, where $\alpha_1 = 2.34$, $\alpha_2 = 4.09$, $\alpha_3 = 5.52$, $\alpha_4 = 6.79$, $\ldots$, for example. Under the condition $s\lambda^4 \ll 1$ and using expressions (2.12) and (2.13), the parameter η_1 can be written in the following explicit forms: for $x > 0$,

$$\eta_1 = p[1 - 2s\lambda^4 x^{-3/2}(x^{1/2} - \tan^{-1} x^{1/2})] \tag{3.9}$$

where

$$p = \frac{\nu s^{1/3} x}{1 + x} \qquad x = \frac{\nu s \lambda^2}{1 - 2s\lambda^4} - 1$$

and where

$$\nu = \frac{d}{2a_0}\left(1 + 2\frac{b}{d}\right)$$

and, for $x < 0$,

$$\eta_1 = -|p|\left[1 + 2s\lambda^4 y^{-3/2}\left(y^{1/2} + \frac{1}{2}\ln\frac{1 - y^{1/2}}{1 + y^{1/2}}\right)\right] \tag{3.10}$$

where $y = -x$.

The quantum numbers λ_k and the 'electric' energy levels $W_k^{(0)}$ can then be written in the explicit form

$$2\frac{W_k^{(0)}}{R} = -1/\lambda_k^2 = -\frac{1}{2}(\nu s - \alpha_k s^{2/3}) - \sqrt{\frac{1}{4}(\nu s - \alpha_k s^{2/3})^2 + \frac{8}{3}s}. \tag{3.11}$$

The last term under the square-root sign describes the effect of the one-dimensional Coulomb field on the levels in the triangular well.

Thus, in the zeroth approximation, the system of energy levels is the sum of two independent series of energies. The first series $W_\lambda^{(0)}$ are the quasi-Coulomb levels (3.7) and (3.8) also influenced by the electric field E. The second series is formed by the 'electric' levels $W_k^{(0)}$ as given by (3.11). The electron having an energy $W_\lambda^{(0)}$ is localized within the impurity well whilst the electron having energy $W_k^{(0)}$ is localized within the triangular well close to the right-hand boundary of the QW. For a sufficiently weak electric field E, the group of the 'electric' levels has energy greater than the quasi-Coulomb group, so the relevant states are not in resonance.

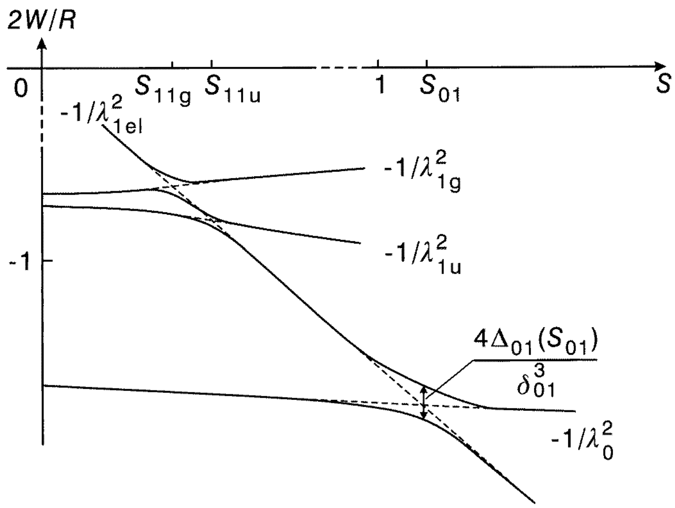

Figure 2. The dimensionless energy $2W/R$ plotted as a function of the dimensionless electric field s (solid lines). The dashed lines display the independent ground 'electric' level ($\lambda_{1,el}$, equation (3.11)) and ground (λ_0, equation (3.7)) and first excited quasi-even (λ_{1g}) and quasi-odd (λ_{1u}) quasi-Coulomb levels (equation (3.8)). The relevant resonance fields s_{nk} and the resonance splitting of the ground quasi-Coulomb and 'electric' levels, where Δ_{01} is defined by (3.15), are indicated.

3.1.3. The resonant electric fields. If the electric field increases in magnitude, the 'electric' levels move toward lower energy values. Under the condition $d/a_0 \gg 1$, the red shift ($\sim -sd/a_0$) of the 'electric' levels is larger than the shift ($\sim -3n^2s/2$) of the components of the excited quasi-Coulomb levels. In turn, this exceeds significantly the red shift ($\sim 5s^2\delta_0^4/16$) of the ground level. Consequently, these two groups of levels can become equal to one another. Under the condition $W_\lambda^{(0)} = W_k^{(0)}$, the relevant quasi-Coulomb and 'electric' levels appear to be in resonance. On using this condition, the magnitudes s_{nk} of the electric field at resonance can be found. These define the cases in which the nth quasi-Coulomb level and the kth 'electric' level are in resonance. The dependencies of the quasi-Coulomb energy levels $W_\lambda^{(0)}$ given in equations (3.7) and (3.8) and the 'electric' energy levels $W_k^{(0)}$ in (3.11) upon the magnitude of the electric field s are displayed in figure 2.

On using the resonance condition for the ground impurity level $\lambda = \delta_0$, where δ_0 is defined by (3.3), the resonance fields s_{0k} are given by

$$s_{0k} = \frac{1}{\delta_0^2 \nu} + \frac{\alpha_k}{\delta_0^{4/3} \nu^{5/3}} - \frac{8}{3\nu^2}. \tag{3.12}$$

For the excited states ($n = 1, 2, \ldots$) the expressions for the resonance fields can be written in the form

$$s_{nk}^{(g,u)} = \frac{1}{\lambda_{ng,u}^2 \nu} + \frac{\alpha_k}{\lambda_{ng,u}^{4/3} \nu^{5/3}} - \frac{8/3 \pm 9/3}{\nu^2} \tag{3.13}$$

where the quantum numbers $\lambda_{g,u} = n + \delta n_{g,u}$ are given by (3.4) and (3.6).

It is seen from figure 1 that, as the impurity centre shifts towards the right-hand boundary of the QW, the parameters b and ν decrease. As a consequence, the resonance fields s_{nk} given by equations (3.12) and (3.13) increase. In turn, the magnitude of the electric field is limited by the condition $s\lambda^4 \ll 1$. We thus obtain the result $2|b|/d \ll 1 - 2a_0\lambda^2/d$, which implies that the above results are applicable for the cases where the impurity is separated from the right-hand edge of the QW by a distance greater than the impurity radius a_0.

3.2. The first approximation

In a first approximation, the last term in the left-hand part of equation (2.12) is taken into account. We expand the Airy functions $\mathrm{Ai}(-\eta_1)$ and the first factor in the first term in (2.15) in a power series in $\lambda - \lambda_k$ and $\lambda - \lambda_n$ respectively, where λ_k and λ_n are obtained in the zeroth approximation. Also we use the explicit expressions for $\eta_1(\lambda)$, $\varphi(\lambda)$, $\xi(\lambda)$ and $Q(\lambda)$ from (3.9), (2.16) and (2.17). On substituting the resulting expansions into equation (2.15) and using both equations (3.1), we arrive at a quadratic equation for the quantum number λ. The roots of this equation can be written in the form

$$\lambda^{(1,2)} = \tfrac{1}{2}(\lambda_n + \lambda_k) \pm [\tfrac{1}{4}(\lambda_n - \lambda_k)^2 + \Delta_{nk}^2(s)]^{1/2} \tag{3.14}$$

where

$$\Delta_{nk}(s) = \beta_{nk}^{1/2} s^{1/3} \mathrm{e}^{-\Phi(s)} \tag{3.15}$$

with

$$\beta_{0k} = -\frac{\mathrm{Bi}(-\alpha_k)[(2|Q| + \xi) - \delta_0^{-1} G_\lambda]\lambda_k^3 \delta_0}{4\,\mathrm{Ai}'(-\alpha_k)(2|Q| + \xi)} \tag{3.16}$$

and

$$\beta_{nk} = -\frac{\mathrm{Bi}(-\alpha_k)[2|Q| + \xi - n(n!)^{-2}\delta_n^{-2} G_\lambda]\lambda_k^3 n}{8\,\mathrm{Ai}'(-\alpha_k)(2|Q| + \xi)n!^2}. \tag{3.17}$$

The δ_n are defined by the expressions (3.3), (3.4) and (3.6) for the quasi-even or quasi-odd components respectively.

Equation (3.14) describes the effect of anti-crossing of the energy levels which are derived from a state which was originally located in different parts of the effective potential. It follows from (3.14) that, if the electric field s and the resonance value s_{nk} are far apart, and hence $\frac{1}{4}(\lambda_n - \lambda_k)^2 \gg \Delta_{nk}^2(s)$, the quantum numbers are close to those obtained in the zeroth approximation, namely $\lambda^{(1)} \cong \lambda_n$ and $\lambda^{(2)} \cong \lambda_k$. In the case of resonance for which $s = s_{nk}$ and $\lambda_n = \lambda_k \equiv \lambda_{nk}$, it is found that $\lambda^{(1,2)} = \lambda_{nk} \pm \Delta_{nk}(s_{nk})$.

At resonance, the differences between the quantum numbers $\lambda^{(1)}$ and $\lambda^{(2)}$ and their associated energies $W_{\lambda^{(1)}} - W_{\lambda^{(2)}} = \Delta W_{nk}$ are given by

$$\lambda^{(1)} - \lambda^{(2)} = 2\Delta_{nk}(s_{nk}) \qquad \text{and} \qquad \Delta W_{nk} = 2R\Delta_{nk}(s_{nk})/\lambda_{nk}^3. \tag{3.18}$$

Thus if resonance between the quasi-Coulomb levels (3.7) and (3.8) and the ‘electric’ levels (3.11) occurs, any crossing occurring in the zeroth approximation turns into anti-crossing in the first approximation.

4. Discussion

The basic physics associated with the electron states has been kept in view in the above analysis. As the effect of the magnetic field and the width of the QW on the impurity energy levels had been studied in detail previously [3–5, 8, 9], we have concentrated here on the influence of the electric field E on the electron states. If the effects of the applied electric field s and the resonance fields s_{nk} are very different in magnitude, the system of energy levels is the sum of independent quasi-Coulomb (3.7), (3.8) and ‘electric’ (3.11) levels.

The wave function is concentrated within either the impurity well or the triangular well close to the left- or right-hand boundary of the QW respectively. In the case of resonance for which $s \cong s_{nk}$, the nth quasi-Coulomb and kth 'electric' states become very close in energy. The relevant gap is defined by (3.18). On ignoring possible relaxation processes, coherent resonant tunnelling between the impurity and triangular wells becomes possible. As a result, a drastic redistribution of the wave function and consequent emission of high-frequency coherent radiation occurs. The wave functions related to the components of the energy doublet attain a twin-peaks configuration. Calculations of the wave functions call for specific consideration. The energy level pattern as a function of the magnitude of the electric field is shown in figure 2.

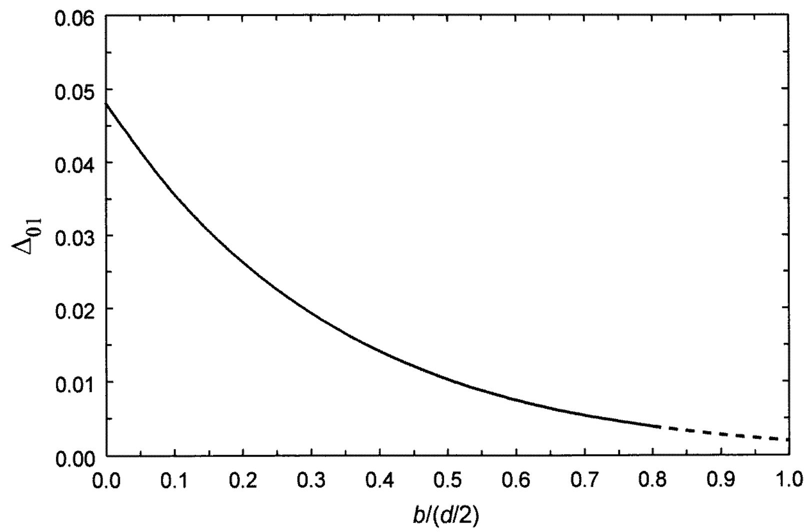

Figure 3. The dependence of the resonance gap Δ_{01} (equation (3.15)) between the ground quasi-Coulomb ($n = 0$) and 'electric' ($k = 1$) levels upon the displacement b of the impurity from the mid-point of the QW. The other parameters are given by $d/a_0 = 6$, $a_B/a_0 = 0.4$, $\delta_0 = 0.52$ and $\beta_{01} = 0.01$.

It is clear from equations (3.15) and (2.13) that the resonance gap Δ_{nk} increases as the resonance field s_{nk} increases. Expressions (3.12) and (3.13) enable the dependence of the resonance field s_{nk} on the 'electric' (k) and quasi-Coulomb (n) indexes, on the width of the QW d and on the position of the impurity centre b to be obtained. It follows that, for a fixed index of the quasi-Coulomb level n, the resonance field s_{nk} increases as a function of the index of the 'electric' level k. Meanwhile, for a fixed index k, the resonance field s_{nk} decreases with increasing index n. The wider the QW, the smaller the resonance field s_{nk}. The shift of the impurity centre towards the right-hand boundary (b decreases) leads to an increase in magnitude of the resonance field s_{nk}. Figure 3 shows the dependence of the resonance gap between the ground quasi-Coulomb and 'electric' levels as function of the position of the impurity.

From the approach described above, the dependencies of the energy W_λ of the longitudinal motion of the impurity electron on the magnitudes of the magnetic and electric fields, on the width of the QW and on the position of the impurity have been obtained. It follows from (3.3), (3.4) and (3.6) that, if the magnetic field increases in magnitude, then the energy W_λ decreases. As the electric field increases, the energy of the ground state (3.7) decreases only slightly ($\sim s^2$). For the excited states (3.8), as the electric field increases the energy of the quasi-even component increases ($\sim +s$), while the energy of the quasi-odd component decreases ($\sim -s$). As the width of the QW decreases and the impurity centre shifts away from the mid-point, the energy W_λ increases in each case.

A comparison of our analytical results with those obtained by numerical methods would be desirable at this point. Cen and Bajaj [6] have extended the variational approach [3, 4] to the calculation of the binding energy of the impurity electron in a 'dielectric' QW (for which the dielectric constant of the barrier material is much less than that of the well material) for the case in which both the magnetic and electric fields are perpendicular to the hetero-planes. As pointed out in reference [6], a difference between the values of the dielectric constants has little effect, allowing a qualitative comparison between the results to be made. For confined systems, the binding energy W_b is given by $W_b = W_0 - W_\lambda$ where W_λ is the energy of the impurity electron and W_0 is the energy of the electron in the QW containing no impurity centre. In the presence of an electric field E, the expression for W_0 can be obtained from (3.11) at $b = 0$ by neglecting the last term under the square-root sign. In the absence of an electric field, expressions for the energy levels in the QW are well known. We thus have, for the ground state,

$$2W_b/R = q + \delta_0^2 + (5/16)s^2\delta_0^4 \tag{4.1}$$

where

$$q = \begin{cases} \pi^2 a_0^2/d^2 & \text{for } E = 0 \\ -\nu s + \alpha_0 s^{2/3} & \text{for } E \neq 0. \end{cases} \tag{4.2}$$

The dependencies of the binding energy upon the magnitude of the magnetic field B and on the displacement of the impurity b are dominated by the term δ_0^{-2} in equation (4.1). It follows from (3.3) and (3.2) that, if the magnetic field increases in magnitude, the binding energy also increases. An impurity located at the mid-point of the QW ($b = 0$) produces the largest binding energy. With increasing distance of the impurity from the centre towards both the left (3.2) and right [8] boundaries of the QW, the binding energy decreases. These results coincide with those obtained numerically by a variational approach [4, 6] and the method of direct integration [19].

The first term $\sim d^{-2}$ in equation (4.1) contributes mostly to the dependence of the binding energy upon the width d of the QW with no electric field. Clearly, the narrowing of the QW leads to an increase in the binding energy. In the presence of the electric field, this dependence becomes more pronounced. These results are in agreement with those found previously [3, 6]. Also, from (4.1) and (4.2), it follows that the binding energy of the impurity centre when it is positioned close to the mid-point ($2b/d < 1$) of the wide QW ($d/a_0 > 1$) decreases if the electric field increases in magnitude. Good agreement is found between our analytical calculations and the variational-type calculations presented in reference [6].

Our results may be compared with those obtained for the problem of an impurity centre in a quantum dot subjected also to a magnetic field. A similar set of results were obtained in reference [20] for the binding energy as a function of the magnitude of the magnetic field, the position of the impurity centre and the size of the quantum dot.

Recently, Cao *et al* [21] have calculated the excitonic states in a superlattice coupled to an enlarged quantum well in the presence of an electric field. For the resonance electric field, the energy levels in the quantum well and the Stark levels in the superlattice were shown to anti-cross. Moreover, the pattern of the energy levels given in reference [21] correlates well with that shown in figure 2. Consequently it is suggested that the anti-crossing of the resonance levels caused by the electric field is similar for various types of low-dimensional structure.

Let us consider possible experiments. Suitable values for the parameters for the GaAs QW are needed for the case of a strong magnetic field for a well in which the width $d \gg a_0$. Thus we take $\mu = 0.067m_0$, $\varepsilon = 12.5$ and $a_0 = 98.7$ Å with $B = 40$ T and $a_B/a_0 = 0.4$. A red shift of the ground impurity state ($n = 0$, $\lambda_0 = \delta_0 = 0.52$, from reference [9]) $\Delta W_0^{(0)}$ caused by the electric field $E = 1200$ kV m^{-1} may be found from (3.7) such that $\Delta W_0^{(0)} = 0.45$ meV. The excited states are more sensitive to the effect of the electric field; for example, for an electric field $E = 30$ kV m^{-1}, the splitting of the first excited impurity level ($n = 1$, $\lambda_{1g,u} = 1.42$, from reference [9]) is $\Delta W_{1g,u}^{(0)} = W_{1g}^{(0)} - W_{1u}^{(0)}$ which gives the result $\Delta W_{1g,u}^{(0)} = 1.95$ meV from (3.8). The resonance splitting of the ground quasi-Coulomb and 'electric' levels ΔW_{01} ($n = 0$, $k = 1$, $\beta_{01} = 0.01$) is defined by (3.18), (3.15) and (2.13) such that $\Delta W_{01} = 1.84$ meV. This gap corresponds to a frequency of 0.44 THz for the emitted radiation. The relevant resonance electric field E_{01} is equal to 710 kV m^{-1} ($s_{01} = 1.23$). These values are those typically found in experiments. The chosen electric field E_{01} causes the penetration through the potential barrier to be relatively weak. This in turn leads to the result that a wide QW width of $d = 600$ Å is needed to demonstrate this effect. When the QW becomes narrower, this value for E_{01} is exceeded, the penetration increases and the above method of solving equation (2.12) becomes inappropriate. However, clearly in the presence of a stronger electric field ($s \leqslant 2$), the effect of the resonance splitting holds for QWs of standard width $d \sim (3\text{–}5)a_0$. In this case, a numerical approach should be used.

5. Conclusions

We have developed an analytical method for solving the problem of an electron (hole) captured by an impurity centre positioned anywhere between the mid-point and edge of a single quantum well in the presence of strong magnetic and electric fields directed perpendicular to the hetero-planes. It has been shown that the combined potential is similar to that of a double quantum well. Resonance between the levels associated with different wells occurs for specific values of the electric field and coherent tunnelling between the wells becomes possible. The single QW considered above exemplifies this resonance structure. The relevant resonance energy levels are found to anti-cross. The dependencies of the impurity levels and their resonance splitting on the width of the well, the position of the impurity and on the magnitudes of the electric and magnetic fields have been obtained. This in turn defines the tunnelling time and the period of inter-well oscillations of the electron and the frequency of the relevant emitted radiation. Estimates of the resonance gap and the frequency of the emitted radiation for GaAs are shown to be in general agreement with experimental values. The results obtained can be extended to QWs containing impurity centres distributed throughout the width of the QW. The presence of these resonance states markedly affects the optical spectra of semiconductor structures containing impurity-doped QWs in the presence of magnetic and electric fields.

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